Quantum Simulation of Slow Light in Photonic Crystals

Introduction

Quantum computing is a computational paradigm that harnesses quantum mechanics to store and manipulate information. Efforts to build universal quantum computers are actively underway in academia and industry, with fast progress being made in recent years. To reliably store quantum information, scaled quantum computers will require a memory system that serves a similar function to classical memory. While the underlying physics of how to build quantum memory systems is not yet understood, slow light phenomena has emerged as a promising avenue for the realization of this technology [1].

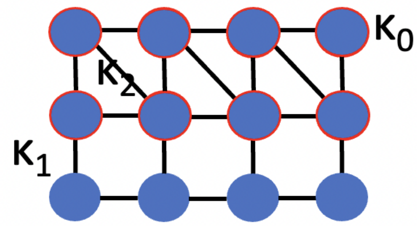

Figure 1: The lattice from [3] Coupling terms $\kappa_0, \kappa_1, \kappa_2$ are in units of the horizontal tunneling amplitudes.

Slow light is a phenomenon where a photon's group velocity approaches zero. Physicists can engineer slow light systems by building devices whose dispersion relations $\omega(k)$ have stationary points. The dynamics of slow light can be studied by coupling qubits to these systems, enabling us to observe time-dependent photon dynamics called non-Markovian physics. An example would be a single photon predictably exciting a qubit, which I investigate here.

Engineering photonic crystals (Figure 1) whose dispersion relations have stationary points is one way to generate slow light. In past experiments studying non-Markovian physics, qubit frequencies have been coupled near band edges [2]; however, this traps photon excitation in bound states so it does not propagate as slow light. To solve this problem, photonic crystals can be engineered to have dispersion relations with stationary points that are also inflection points (Figure 2) so that $\omega'(k) = \omega''(k) = 0$. Slow light generated by coupling superconducting qubits to photonic crystals at stationary inflection points (SIP) in the frequency domain has yet to be studied. Thus, in my project, I computationally simulate a lattice with SIPs [3] to determine if it can be used as an experimental quantum simulation device to induce non-Markovian photon dynamics.

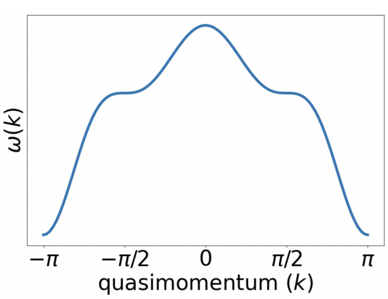

Figure 2: The dispersion relation for the lattice. Stationary inflection points occur at $k = -\pi/2, \pi/2$.

Methods

The Hamiltonian of this lattice is expressed by the tight-binding model [3]. The probability that a photon is on a node at a given time is given by the squared modulus of the field amplitudes E_l expressed on the left side of (1).

$$\begin{aligned} \omega E_l^{(1)} &= E_{l-1}^{(1)} + E_{l+1}^{(1)} + \kappa_1 E_l^{(2)} + \kappa_2 E_{l+1}^{(2)} + \kappa_0 E_l^{(1)} \\ \omega E_l^{(2)} &= E_{l-1}^{(2)} + E_{l+1}^{(2)} + \kappa_1 \left(E_l^{(1)} + E_l^{(3)} \right) + \kappa_2 E_{l-1}^{(1)} + \kappa_0 E_l^{(2)} \\ \omega E_l^{(3)} &= E_{l-1}^{(3)} + E_{l+1}^{(3)} + \kappa_1 E_l^{(2)} \end{aligned} \quad (1)$$

κ_0 represents on-site energy, and $\kappa_{1,2}$ represent coupling strengths between nodes in terms of the horizontal couplings shown in Figure 1. To obtain the photon dynamics, I used the Schrodinger picture $d\mathbf{E}/dt = -iH\mathbf{E}$, where $\mathbf{E}$ is a vector of field amplitudes and H is the coefficient matrix for the right side of (1). I solved the differential equation to get the time-dependent equation for field amplitudes

$$\mathbf{E}(t) = \exp(-iHt)\mathbf{E}_0. \quad (2)$$

Results

To confirm that the photon dynamics show non-Markovian physics, I used equation (2) to program a time-evolution simulation of this lattice coupled to a qubit at the SIP frequency. I picked a specific set of values for $\kappa_{0,1,2}$ that produce the symmetric SIPs seen in Figure 2. My simulation's results are shown in Figure 3, with markers indicating four important steps. **1.** The photon begins as a qubit excitation and evolves according to equation (2). **2.** Immediately after $t = 0$, the probability that the qubit is excited decays (approximately) exponentially. There is a high probability in this initial decay that the photon has tunneled to the qubit's nearest-neighbor lattice site. **3.** The qubit then predictably experiences a re-excitation with ~30% probability. Critically, this revival strongly suggests that photon dynamics are non-Markovian; otherwise, if each Δt were time-independent, the decay would be completely smooth. **4.** Finally, qubit excitation decays again, and the probability of finding the photon spreads out across the lattice.

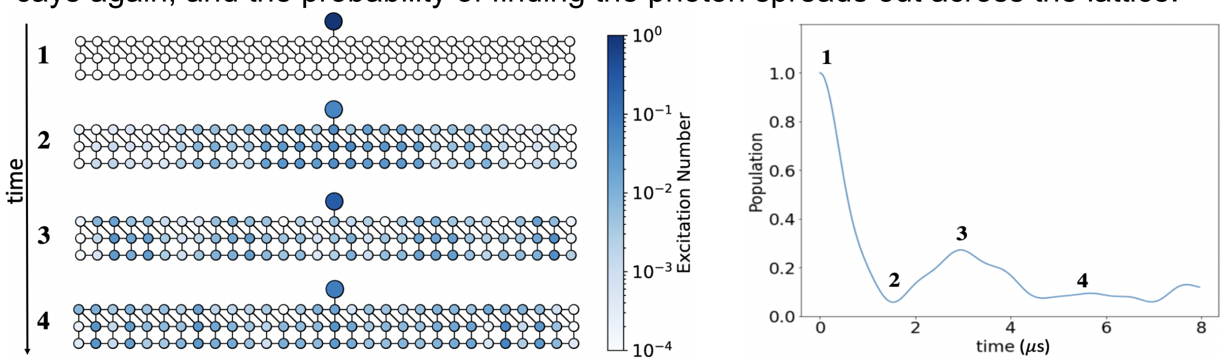

Figure 3: (a) Time-evolution simulation of 30 unit-cell lattice coupled to qubit. (b) Population graph.

To ensure this re-excitation behavior is due to non-Markovian physics (and not other phenomena), I first verified the photon is indeed slow-moving. I moved the qubit *away* from the SIP frequency and reran my simulation (Figure 4a), finding that qubit excitation decays at the ns timescale as opposed to the $\mu\mathrm{s}$ timescale that we saw in Figure 3. Photon group velocity is several orders of magnitude faster than in the previous scenario, indicating that slow light is not present when the qubit is moved away from the SIP frequency. This confirms the photon *is* slow-moving in Figure 3.

Next, I verified that the qubit re-excitation at marker **3** is due to slow light physics, not a spurious excitation caused by photon reflection from lattice edges. In Figure 4b, I ran the same experiment as Figure 3, but incorporated dissipation at end nodes. Dissipation specifies the rate at which the photon exists the system when it reaches the lattice edge. If the photon does not exit the system, it could traverse back to the qubit and cause a spurious excitation. Increasing dissipation decreases the magnitudes of all but the first peak, indicating that only this peak represents a slow light re-excitation independent of reflection. If we decrease lattice size, reflection peaks interact more with revival peaks. After testing many lattice sizes, I found that no lattice smaller than 30-unit cells shows consistent non-Markovian revivals independent of reflections. This puts a lower bound on the size of photonic crystals that we could implement experimentally.

However, physical devices will inevitably be non-ideal. Thus, I added a disorder term sampled from a Gaussian distribution to all couplings, node frequencies, and qubit frequencies. I found non-Markovian revivals could be reliably achieved with a standard deviation up to 2 MHz (Figure 4c). This is approximately the average device disorder for realistic superconducting qubits and lumped element resonators (which will be nodes in

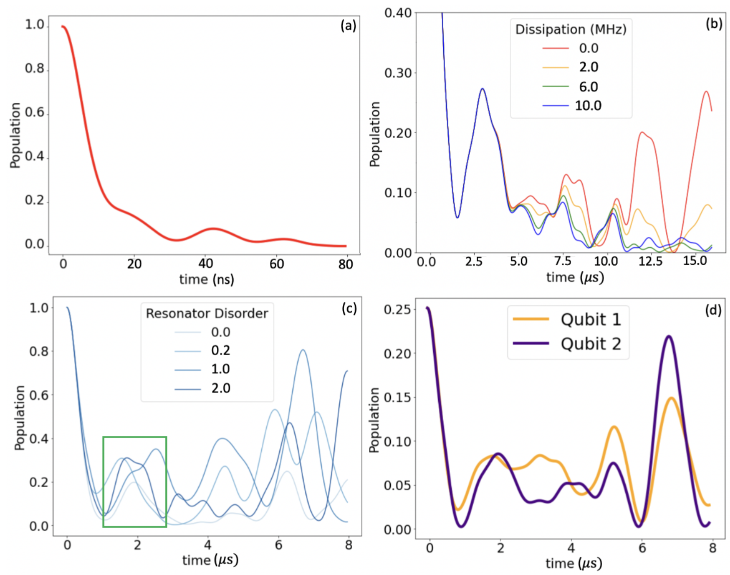

Figure 4: (a) Qubit frequency off SIP. (b) Effect of dissipation on reflection. (c) Effect of disorder on re-excitation. (d) Two-qubit.

the physical lattice). In Figure 4c, the consistent first peak around $2\mu s$ present for all disorder values indicates slow light revivals similar to Figure 3. This is the most important result providing confidence that the experimental implementation of this device will successfully produce non-Markovian dynamics, allowing us to study slow light. Profoundly, my model showed that superconducting photonic crystals are robust enough to produce slow light even with current disorders for transmon qubits and resonators.

To utilize slow light for quantum memory, it must be generated simultaneously on more than one qubit. To test the scalability of this regime for multiple qubits, I simulated the lattice from Figure 3a coupled to an additional qubit placed one unit-cell away from the first qubit (Figure 4d). I found that both qubits exhibit non-Markovian revivals at approximately $2\mu\text{s}$ confirming that we may scale this slow light system over numerous qubits.

Conclusion and Future Research

My simulation indicates that we can induce non-Markovian behavior by coupling a qubit to a lattice at an SIP frequency, a novel setup that had not been studied before. Most importantly, my simulation shows that even in weakly disordered systems, we can still engineer slow light behavior in photonic crystals. This provides proof-of-concept that building this device will allow us to physically study slow light.

Moving forward, I plan to implement this quantum simulation device by physically building the photonic crystal on a superconducting circuit: I will place lumped-element resonators on a chip arranged like the lattice from [3] and capacitively couple it to a superconducting transmon. Then, I can take time-domain measurements from this system in a dilution fridge to detect qubit re-excitation behavior. Experimentally confirming that this device reliably induces non-Markovian physics would show that it is a quantum simulator for slow light. Successfully implementing this device allows us to pursue our ultimate goal of studying slow light physics for its use in a quantum memory system.

During this research project, I learned how theorists can work with experimentalists to show device efficacy before beginning physical implementation. Eventually, I plan on collaborating with experimentalists to explore new methods for quantum simulation and develop quantum computer architecture components like quantum memory.

References

[1] A. I. Lvovsky et al. “Optical quantum memory,” *Nature Photonics* **3**, 706–714 (2009).

[2] V. S. Ferreria, et al. “Collapse and Revival of an Artificial Atom Coupled to a Structured Photonic Reservoir,” *PRX* **11**, 041043 (2021).

[3] H. Li, et al. “Frozen Mode Regime in Finite Periodic Structures,” *PRB* **96**, 180301 (2017).